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Probability & Statistics

$$\text{Probability} = \frac{\text{fav. outcome}}{\text{Total outcome}}$$

Types of Events :-

⇒ Sample space (S) = The set of all possible distinct outcome (events) for a random experiment is called Sample space (or even space) providing

- No two or more of these outcomes can occur simultaneously
- Exactly one of the outcomes must occur, whenever the experiment is performed.

☆ Event types :-

- mutually exclusive event
- collictively exclusive event
- independent and dependent event
- Compound event
- Equally likely event
- complimentry events

- Mutually exclusive event :- If two or more event cannot occur simultaneously in a single time of any expriment. Then such events are called mutually exclusive events

- Collectively exclusive events :- A list of events

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is said to be collectively exclusive when all possible events that can occur from an experiment include every possible outcomes

ex :- {$A_1, A_2, \ldots A_n$}

$S = \{A_1 \cup A_2, \ldots\}$

→ Independent & dependent events :- Two events are said to be independent if information about one tells nothing about the occurance of the others

OR

Outcome of one event doesn't affect and is not affected by the other event. The outcome of the successive tosses of a coin are indepen of its preceding toss.

→ Compound events :- When two or more events occurs in collection with each other, then their simultaneous occurances is called a compound event. These events may be depende & independent.

→ Equally likely event :- Two or more events said to be equally likely if each has an equal chance to occur i.e, one of them c be expected to occur in prefarance to t other.

→ Complimentary events :- If 'E' is any sub of the sample space then its complement

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denoted by '$\bar{E}$' contains all the elements of the sample space that are not the part of 'E'

$\boxed{\bar{E} = S - E}$ [all sample elements not in E]

Classical Approach :-

This approach is based on assumption that all the possible ~~elem~~ outcomes (finite in NO.) of any experiment are mutually exclusive & equally - likely. It states, that during the random experiment if there are "a" possible outcomes where the favourable events "A" occur and "b" outcomes where the event "A" doesn't occur and all these possible outcomes are mutually exclusive, exhaustive, equiprobable then the probability that event "A" be defined as $P(A) = a/a+b = n(a)/n(s)$

Fundamental rules of probability

- Each probability should fall b/w 0 and 1 that is $0 \leq P(A_1) \leq 1$ for all i where $P(A_i)$ is read as "probability of event (A_i)"

The probability of a event is restricted to the range 0 to 1 inclusive where "0" represent an impossible event "1" represent a certain event

- $P(S) = P(A_1) + P(A_2) \ldots\ldots + P(A_n) = 1$

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where P(S) is read as "Probability of a certain event. This rule states that the sum of probability of all simple events constituting the sample space is equal to 1. This also implies that if a random experiment is conducted, one of its outcome in sample space is certain to occur.

- If events A_1 and A_2 are two elements in S and if occurance of A_1 implies that A_2 occurs i.e, if A_1 is subset of A_2 then the probability of A_1 is less than or equal to the probability of A_2 that is
$P(A_1) \leq P(A_2)$

- $P(\bar{A}) = 1 - P(A)$ that is the probability of an event that doesn't occur is equal to 1 - the probability of the event that does occur.
(The probability rules for the compliment-any event)

Numericals

Ques 1) Three unbiased coins are tossed what is the probability are
(i) All heads (ii) Two heads (iii) one head
(iv) atleast one head (v) atleast two head
(vi) all tails

Sol:

HHH	HHT	HTH	HTT
THH	THT	TTH	TTT

Total No. of outcomes = 8

(i) Probability of getting all heads :- $\frac{1}{8}$

(ii) Probability of getting two heads :- $\frac{3}{8}$

(iii) Probability of getting one head :- $\frac{3}{8}$

(iv) Probability of getting atleast one head :- $\frac{7}{8}$

(v) Probability of getting atleast two head :- $\frac{4}{8} = \frac{1}{2}$

(vi) Probability of getting all tails :- $\frac{1}{8}$

Combinations :- This counting rules for combinat... allows you to select r (says no. of outcomes) from collection of n distinct outcomes withc... carrying in what order they are arranged. This rule is denoted by

$$C(n, r) = {}^nC_r = \frac{\angle n}{\angle r \angle n-r}$$

where $\angle n = n(n-1)(n-2) \ldots 3\ 2\ 1$ and $\angle 0 =$

⇒ IMP rules.
- ${}^nC_r = {}^nC_{n-r}$, ${}^nC_n = 1$
- if n objects consist of all n_1 of one type, a... n_2 of another type & so on upto n_k all of the kth type, then the total no. of

selection that can be made of 1, 2 3 upto n object is $(n_1+1)(n_2+1)\ldots(n_k+1)-1$
The total no. of selection from n object of all diff. is 2^{n-1}

ues 2) A bag contain 6 red and 8 green balls
(i) if one ball is drawn at random then what is the probability of the ball being green.
(ii) if 2 balls are drawn at random then what is the probability that one is red and other is green

on red balls = 6
green balls = 8
Total balls = 14

) $^{14}C_1 = \frac{14!}{13!} = 14$ //

$\left\{ P(A) = \frac{C(A)}{C(S)} \right\}$

for green :- $^8C_1 = 8$ //

Probability ⇒ $\frac{8}{14}$

$^{14}C_2 = \frac{14!}{2\ 12!} = \frac{^7\cancel{14}\times 13}{\cancel{2}} = 91$ //

$^8C_1 \times {}^6C_1 = 8\times 6 = 48$

Probability of getting 1 red and 1 green ball $= \frac{48}{91}$ //

Ques 3) Tickets are numbered from 1 to 100. They are well shuffled and a ticket is drawn at random what is the probability that the drawn ticket has
(i) an even no.
(ii) The number 5 or multiple of 5
(iii) a number which is greater than 75
(iv) a number which is a square.

son Total outcome = 100

(i) $\frac{50}{100} = \frac{1}{2}$

(ii) $\frac{20}{100} = \frac{1}{5}$

(iii) $\frac{25}{100} = \frac{1}{4}$

(iv) $\frac{10}{100} = \frac{1}{10}$

Ques4) 5 men in a company of 20 are gradua[te]. if 3 men picked out of the 20 at rand[om]
(i) What is the probability that they are all graduate
(ii) atleast one graduate
(iii) No graduat

soln (i) $\frac{^5C_3}{^{20}C_3} =$

$^5C_3 = \frac{5!}{3!\ 2} = \frac{5\times\cancel{4}^2}{\cancel{2}} = 10$

$^{20}C_3 = \frac{20!}{3!\ 17!} = \frac{\cancel{20}\times 19\times \cancel{18}^6}{\cancel{3}\times\cancel{2}} = 11$[...]

$\Rightarrow \frac{10}{1140} = \frac{1}{114}$

iii) No graduate $= \frac{^{15}C_3}{^{20}C_3}$

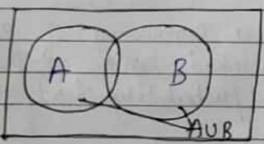
$= \frac{\frac{15 \times 14 \times 13}{3 \times 2}}{\frac{20!}{3!\,17!}} = \frac{455}{\frac{20 \times 19 \times 18}{3 \times 2}}$

No graduates $= \frac{455}{1140}$

Rules of probability and algebra of event

→ Rules of addition

1) Mutually exclusive event:-



If two events A and B are mutually exclusive and exhaustive and equiprobable then the probability of either event A or B or both occurring is equal to the sum of their individual probabilities

$P(A \text{ or } B) = P(A \cup B) = \frac{n(A \cup B)}{n(S)}$

$= \frac{n(A)}{n(S)} + \frac{n(B)}{n(S)} = \frac{n(A) + n(B)}{n(S)}$

$= P(A) + P(B)$

Let 'A' be any event and '$\bar{A}$' be the compliment of A. Obviously A and $\bar{A}$ are mutually exclusive and exhaustive event. Thus either A occurs or it does not, is given by

$\Rightarrow P(A \text{ or } \bar{A}) = P(A) + P(\bar{A}) = P(A) + (1 - P(A)) = 1$

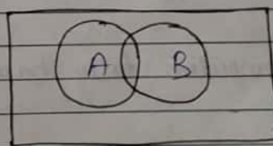
$\Rightarrow P(A) = 1 - P(\bar{A})$

2) Partially overlapping or joint event:

If event A & B are not mutually exclusive, it is possible for both events to occur simultaneously. This means these events have some sample points in common. Such events are also called Joint (or overlapping events)

$P(A \text{ or } B) = P(A) + P(B) - P(A \text{ and } B)$

$P(A \cup B) = P(A) + P(B) - P(A \cap B)$



Ques) Of 1000 assembled component 10 have a working defect & 20 have a structure defect. There is a Good reason to assume that no component has both defect. What is the probability that randomly chosen component will have either type of defect

Sol $P(A) = \frac{10}{1000}$, $P(B) = \frac{20}{1000}$

$P(A \cup B) = P(A) + P(B) - P(A \cap B)$

$0.01 + 0.02 - 00 = 0.03$

Ques) The probability that a contractor will get a plumbing contract is 2/3 and the probability that he will not get an electrical contract is 5/9 if the probability of getting atleast one contract is 4/5

(i) what is the probability that he will get both

Sol (i) $P(A \cup B) = P(A) + P(B) - P(A \cap B)$

$P(A \cup B) = \frac{2}{3} + \frac{5}{9} - \frac{4}{5} = \frac{18+15}{27} - \frac{4}{5}$

$P(A \cup B) = \frac{33}{27} - \frac{4}{5} = \frac{165-108}{135} = \frac{57}{135}$

$P(A \cup B) = \frac{19}{45}$

Ques) from a computer tally based on computer

Sol i) $P(A \cup B) = P(A) + P(B) - P(A \cap B)$

$P(A \cup B) = \frac{15}{100} + \frac{25}{100} - \frac{5}{100}$

$P(A \cup B) \quad \frac{40-5}{100} = \frac{35}{100} = 0.35$

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Probability Distribution

A listing of all the possible of random variable with each outcome associated with probability of occurrence is called Probability distribution

The numerical value of Random variable depends upon the outcomes of an experiment and may be diff. trials of the ~~exam~~ experiment. The set of all such values so obtained is called Range space of the Random Variable

Ques) If a coin is tossed twice then same space of events, for this random experiment is $S = \{HH, TH, HT, TT\}$

Sol^n $S = \{HH, TH, HT, TT\}$
Head occurs $= 2 \quad 1 \quad 1 \quad 0$

sample space $= \{0, 1, 2\}$

⇒ Types of PD :-

→ Binomial Distribution (BD) :-
Binomial distribution is a widely used PD for a ~~distr~~ discrete random variables for each trial of an experiment. There are only two possible complimentary (mutually exclusive) outcomes such as defective or Good, head or tail, 0 or 1, Boy and Girl, in such case the outcomes of interest is referred to as a "success" and the other as "failure"

→ Properties of BD :-

It is a discrete distribution
It is applied when the event of all trials are independent
The result of each trial can be classified in 2 categories :-
(i) Success (ii) failure

4. Probability formula / function :-

$$P(x = r \text{ success}) = {}^nC_r\, p^r q^{n-r} = \frac{\lfloor n}{\lfloor r \lfloor n-r}\, p^r q^{n-r}$$

where $r = 0, 1, 2, 3 \ldots n$
n = No. of trials or sample size
p = probability of success
$q = (1-p)$, prob of failure
x = discrete binomial random variable
r = No. of the success in n trials.

★ Characteristics of BD :-

1. The mean and standard deviation of a binomial distribution are computed in a shortcut method as follows :-
(i) Mean (μ) :- $\mu = np$
Standard declaration $\sigma = \sqrt{npq}$
Knowing the value of first two central moments

⇒ $\mu_0 = 1$ and $\mu_1 = 1$
⇒ Second moment :- $\mu_2 = npq$
⇒ Third moment :- $\mu_3 = npq(q-p)$
⇒ $\mu_4 = 3n^2p^2q^2 + npq(1-6pq)$

Coefficient of skewness :- $\gamma_1 = \sqrt{\beta_1} = \frac{\mu_3}{\mu_2^{3/2}}$
$= \frac{q-p}{\sqrt{npq}}$

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Where $\beta_1 = \frac{n^2 p^2 q^2 (q-p)^2}{n^3 p^3 q^3}$

Coefficient of Kurtosis :

$\gamma_2 = \beta_2 - 3 = \frac{\mu_4}{\mu_2^2} - 3 = \frac{1 - 6pq}{n - pq}$

Where $\beta_2 = \frac{3n^2 p^2 q^2 + npq(1-6pq)}{n^2 p^2 q^2}$

Ques) 5 coins are tossed, find the prob. of getting 3 Heads

Soln $n = 5, \; p = 1/2, \; q = 1/2, \; r = 3$

$\boxed{P(x) = {}^nC_r \, p^r q^{n-r}}$

$P(x) = {}^5C_3 \left(\frac{1}{2}\right)^3 \left(\frac{1}{2}\right)^{5-3}$

$P(x) = \frac{5!}{3!\,2!} \cdot \frac{1}{8} \cdot \frac{1}{4}$

$P(x) = \frac{5 \times \cancel{4}^{2} \times \cancel{3!}}{\cancel{3!} \times \cancel{2}} \cdot \frac{1}{8} \cdot \frac{1}{\cancel{4}2}$

$P(x) = \frac{5}{16}$ ans//

Ques) A brokerage survey reports that 30% of individual investor have used a discount broker this is one which does not charge the full comission. In a random sample of 9 individual, what is the prob. that

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(i) Exactly two of the sampled individual have used a discount broker

$P = 30/100 \Rightarrow P = 0.30$

$q = 1 - P \Rightarrow q = 0.70$

$n = 9$

$P(X=2) = {}^9C_2 (0.30)^2 (.70)^5$

$P(X=2) = \frac{9!}{2!\,5!} \; 0.09 \times 0.16807$

$\frac{9 \times \cancel{8}^{4} \times 7 \times 6}{\cancel{2}}$ ~~22.27~~ 0.014

$\Rightarrow 0.26$

(ii) Not more than three have used a discount broker

$P(X \le 3) = P(X=0) + P(X=1) + P(X=2) + P(X=3)$

$P(X=0) = {}^9C_0 (0.30)^0 (0.70)^9$

$= \cancel{0.09 \times 0.168 \times (0.70)^4} \quad 1 \times 0.040$

$= \cancel{0.0403} \quad 0.040$

$P(X=1) = {}^9C_1 (0.30)^1 (0.70)^7$

$= 9 \times 0.30 \times (0.70)^7$

$= 0.1556$

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Ques 6.9) Mr. Gupta applies for a personal loan of Rs 150,000 from a nationalist bank to repair house. The loan offers informed him that over the years, bank has received about 2920 loan application per year and that the probability of approval was, on avg. above 0.85

(i) Mr. Gupta wants to know the avg and standard deviation of the number of loan approved per year

Soln
$\Rightarrow \mu = np = 2920 \times 0.85$

$\Rightarrow \mu = 2482$

$\Rightarrow \sigma = \sqrt{npq} = \sqrt{2920 \times 0.85 \times 0.15} = 19.295$

ii) Suppose bank actually received 2654 loan application per year with an approval probability of 0.82. What are the mean and standard deviation now?

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Soln
$\Rightarrow$ mean $= np = 2654 \times 0.82$

$\Rightarrow$ mean $= 2020.48$

$\Rightarrow$ Standard deviation $(\sigma) = \sqrt{npq}$

$\Rightarrow \sigma = \sqrt{2654 \times 0.82 \times 0.18} = \sqrt{391.7304}$

$\Rightarrow \sigma = 19.7921$

Ques 6.10) Suppose 10% of new scooters will require warranty service within the first month of its sale. A scooter manufacturing company sells 1000 scooters in a month

(1) find the mean and standard deviation of scooters that require warranty service

(ii) Calculate the moment coefficient of skewness and kurtosis of the distribution

Soln
$n = 1000$, $p = 0.1$, $q = 1 - p = 0.9$

i) $\Rightarrow$ mean $= np = 100\cancel{0} \times \frac{1}{1\cancel{0}}$

$\Rightarrow$ mean $= 100$

$\Rightarrow$ Standard deviation $(\sigma) = \sqrt{npq}$

$\Rightarrow \sigma = \sqrt{100\cancel{0} \times \frac{1}{1\cancel{0}} \times \frac{9}{1\cancel{0}}} = \sqrt{90}$

$\Rightarrow \boxed{\sigma = 9.46 = 10 \text{ approx}}$

ii) Skewness $= \gamma_1 = \sqrt{\beta_1} = \frac{q-p}{\sqrt{npq}}$

$\Rightarrow \gamma_1 = \frac{0.9 - 0.1}{10} = \frac{0.8}{10} \Rightarrow \boxed{\gamma_1 = 0.08}$

$\Rightarrow \gamma_2 = \frac{1 - 6pq}{npq} = \frac{1 - 6 \times 0.1 \times 0.9}{1000 \times 0.1 \times 0.9}$

$$Y_2 = \frac{1 - 6\times 0.09}{90} = \frac{1 - 0.54}{90} = \frac{0.46}{90}$$

$\Rightarrow Y_2 = 0.0051$

Ques 6.11) The normal rate of infection of a certain disease in animal is known to be 25 per cent. In an experiment with 6 animals injected with a new vaccine it was observed than none of the animal caught the infection. Calculate the probability of the observed result

Sol^n $n=6,\ p = \frac{25}{100} = 0.25,\ r=0$

$q = 1-p = 1-0.25 = 0.75$

$P(X=0)\ {}^nC_r\, p^r\, q^{n-r}$

$\Rightarrow {}^6C_0\ (0.25)^0\ (0.75)^6$

$\Rightarrow 1\times(0.75)^6$

$\Rightarrow 6.17797$

Poission Distribution

The poission distribution process measures the no. of occurance of a particular outcome of a discrete random variables in a predefined time interval space or volume for which an avg. no. of occurance of the outcomes are know or can be determined in the poisson process the random values needs counting. It is applied when no of trials are very large & probability of success is very small.

$$P(X=r) = \frac{\lambda^r e^{-\lambda}}{\angle r}$$

r = required no of success

λ = mean or avg.

e = exponential constant $= 2.7183$

$e^{-1} = 0.36788$, $e^{-3} = 0.04979$, $e^{-5} = 0.06737$

$e^{-2} = 0.13534$, $e^{-4} = 0.01831$

Characterstics of Poisson Distribution (PD)

The arithmetic mean $\mu = E(X)$ of PD is Given by

$$\mu = \Sigma X\, P(X) = \Sigma X\ \frac{e^{-\lambda}\cdot\lambda^x}{\angle X}$$

$$= 0 + \lambda e^{-\lambda} + \lambda^2 e^{-\lambda} + \frac{\lambda^3}{\angle 2} e^{-\lambda}$$

$$= \lambda e^{-\lambda}\left[1 + \lambda + \frac{\lambda^2}{\angle 2} + \dots \frac{\lambda^{r-1}}{\angle X-1}\right]$$

$$\lambda e^{-\lambda} e^{\lambda} = \lambda e^{-\lambda+\lambda}$$

$$\lambda e^{-0}$$

$$\lambda$$

$\sigma^2 = \lambda = np$

$\mu_2 = \mu_3 = \lambda$ and $\mu_4 = \lambda + 3\lambda^2$

$\Rightarrow$ Coefficient of Skewness :-

$$Y_1 = \sqrt{B_1} = \frac{1}{\sqrt{\lambda}}$$

Ques) The number of customers appear at the ticket counter of PVR theatre at a rate of 120 per hour. Find the probability of
(a) No customer appears, (b) only one customer appears, (c) only two customer appears, (d) only three customer appears, (e) at least two customer appears, (f) more than two customers appear (g) b/w one and three customers (both inclusive) appear (h) at the most two customers appears, and (i) less than three customer appears

Sol: $P(r) = \dfrac{e^{-m} m^r}{\angle r}$

Avg no. of customers appearing per min $= \dfrac{120}{60}$

$\Rightarrow m = 2$

$P(r) = \dfrac{e^{-2} 2^r}{\angle r}$

(a) $P(r=0) = \dfrac{e^{-2} 2^0}{\angle 0}$ $\{\because e^{-2} = 0.1354\}$

$\Rightarrow 0.1354$//

(b) $P(r=1) = 0.1354 \times 2 = 0.2708$

(c) $P(r=2) = \dfrac{0.1354 \times 2^2}{2} = 0.2708$

(d) $P(r=3) = \dfrac{0.1354 \times 2^3}{3 \times 2} = 0.180$

(e) $\Rightarrow 1 - P(0) + P(1)$

$\Rightarrow 1 - 0.1354 + 0.2708$

$\Rightarrow 1 - 0.4062$

$\Rightarrow 0.5938$//

(f) $\Rightarrow 1 - [P(r=0) + P(r=1) + P(r=2)]$

$\Rightarrow 1 - [0.1354 + 0.2708 + 0.2708]$

$\Rightarrow 1 - 0.677 \Rightarrow 0.323$

(g) $\Rightarrow P(r=1) + P(r=2) + P(r=3)$

$\Rightarrow 0.2708 + 0.2708 + 0.180$

$\Rightarrow 0.7216$

(h) $P(r=0) + P(r=1) + P(r=2)$

$\Rightarrow 0.1354 + 0.2708 + 0.2708$

$\Rightarrow 0.676$

(i) $P(r=0) + P(r=1) + P(r=2)$

$\Rightarrow 0.1354 + 0.2708 + 0.2708$

$\Rightarrow 0.676$

Ques) Bharat Ltd manufacture blades of which one fifth percentage turn out to be defective. The blades are packed in case each containing 1000 blades. A wholesaler purchase 2000 such case. In how many of them (approx) he may expect to have:
(a) no defective (b) only one defective (c) only two defective (d) only three defective (e) at least two defective (f) ~~only~~ more than two defective (g) b/w one and three defectives (both inclusive) (h) at the most two defectives and (i) less than three defective

Sol: $N = 2000$, $n = 1000$, $P = \dfrac{1}{5} \times \dfrac{1}{100} = \dfrac{1}{500}$

$m = np = 1000 \times \dfrac{1}{500} \Rightarrow m = 2$

$$P(r) = \frac{e^{-m} m^r}{\underline{r}}$$

(a) $P(r=0) = \frac{e^{-2} 2^0}{\underline{0}} = 0.1354$

$\Rightarrow N \times 0.1354 \Rightarrow \frac{\cancel{2000} \times 0.135}{\cancel{1000}}$

$\Rightarrow 270.6$

(b) $N \times P(r=1) = \frac{2000 \times e^{-2} 2^1}{\underline{1}} = \frac{\cancel{2000} \times 0.135 \times 2}{\cancel{1000}}$

$\Rightarrow 540$

(c) $N \times P(r=2) = \frac{2000 \times e^{-2} 2^{\cancel{2}}}{\cancel{2}}$

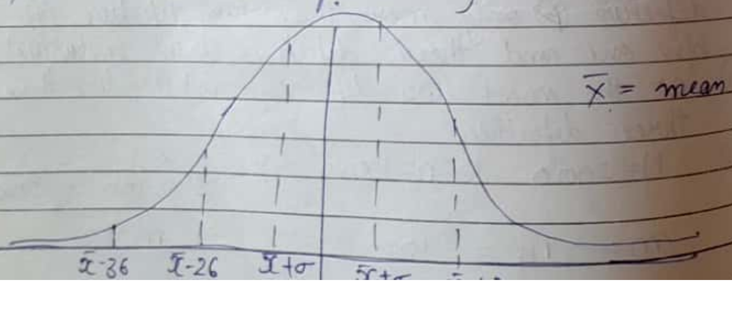
$\Rightarrow 540$

(d) $N \times P(r=3) = \frac{2000 \times e^{-2} \times 2^{\cancel{3}}}{3 \times \cancel{2}} = \frac{2 \times \overset{45}{\cancel{135}} \times 4}{\cancel{3}}$

$\Rightarrow 360$

NORMAL DISTRIBUTION

It is also called Normal Probability distribution, is a continuous probability distribution



$\bar{x}$ = mean

Probability function or Density function $P(X) = \frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{1}{2}\left(\frac{x-\mu}{\sigma}\right)^2}$

$$P(z) = \frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{1}{2}(z^2)}$$

$$z = \frac{x-\mu}{\sigma}$$

z = standard value of x
σ = standard deviation

Ques) A normal distribution has a standard deviation of 200 and mean of 800. find the area of standard normal variate in each of the following cases:-

Sol^n (a) for X = 600

$z = \frac{X-\mu}{\sigma}$ ∵ σ 200, μ = 800

$z = \frac{600-800}{200} \Rightarrow z = -1$

area b/w z = 0 and z = -1 is 0.3413

(b) for X = 700, μ = 800

$Z = \frac{700-800}{200} = \frac{-1\cancel{00}}{2\cancel{00}} = -0.5$

Z = -0.5

area b/w z = 0 to z = 0.5 is 0.1915

(c) for X = 900 μ = 800

$z = \frac{900-800}{200} = 0.5$

z = 0 to z = 0.5 is 0.1915

(d) for $x = 1000$

$z = \frac{1000 - 800}{200} \Rightarrow z = 1$

$z = 0$, $z = 1$ is 0.3413

(e) for x below 600

$x = 600$

$z = \frac{600 - 800}{200} = -1$

$z = -1 - 0.5 \Rightarrow z = -1.5$

$\Rightarrow z = 0.4332$

(f) for x above 700

$x = 700$

$z = \frac{700 - 800}{200} = -\frac{100}{200} = -0.5$

$z = -0.5 + 0.5 = 0$

$\Rightarrow z = 0$

(g) for x below 900

$x = 900$

$z = \frac{900 - 800}{200} = 0.5$

$z = 0.5 - 0.5 = 0$

$z = 0$

(h) for x above 1000

$z = \frac{1000 - 800}{200} = 1$

$z = 1 + 0.5 = 1.5$

$z = 0.4332$

Q) The marks of 1000 students in an exam are found to be normally distributed with mean = 70 & standard declaration = 5 estimate the no. of student

Estimate the No. of student whose marks will be

i) b/w 60 & 75   ii) more than 75

Soln $N = 1000$, $M = 70$, $\sigma = 5$

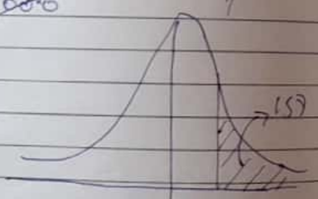
$\Rightarrow z = \frac{60-70}{5} = \frac{-10}{5} = -2 = 0.4772$

$\Rightarrow z = \frac{75-70}{5} = 1 = 1 = 0.3413$

$\Rightarrow z = 0.4772 - 0.3413$

$\Rightarrow z = \cancel{0.1359}\ 0.8185$

$\Rightarrow N \times z = \frac{1000 \times 0.8185}{1000} = 819$



$\Rightarrow 0.5 - 0.3413$

$\Rightarrow -1.587 \times 1000$

$\Rightarrow 158$

Hypothesis

$1\% = \pm 2.58$, $5\% = \pm 1.96$, $10\% = \pm 1.65$

Null hypo. (H_0)

Alternative (H_1)

$H_0\ \mu = 200$ ml

$H_1\ \mu \neq 200$ml

$H_1\ \mu < 200$ml

Quantitative statements about Population

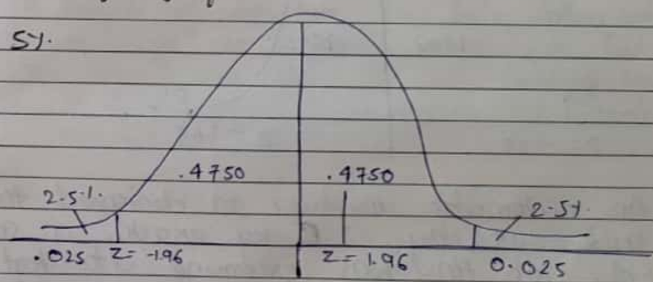
→ (H_0) Null Hypothesis :- It is a claim or statement about population parameter that is assumed to be true until it is declared to be false

→ Alternative Hypothesis (H_1) :- Any hypothesis which is complimantor to null hypothesis (it is also called research hypothesis)

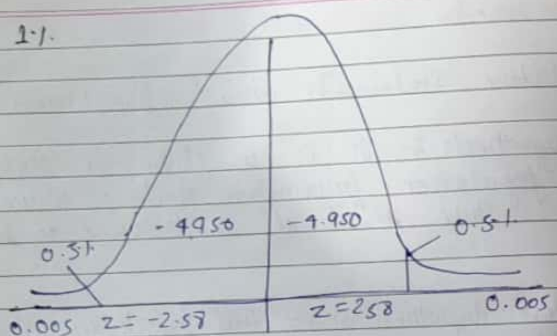
$$z\text{-test} = \frac{x - \mu}{\sigma}$$

⇒ Level of significance :-

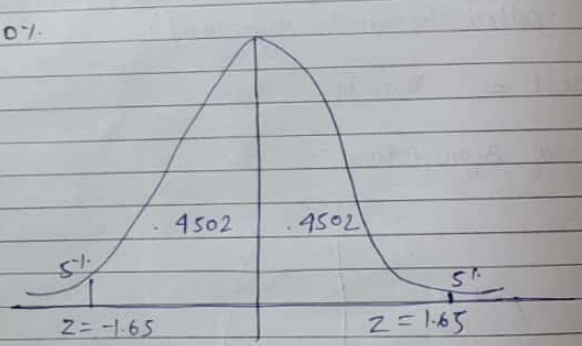
1) 5%



② 1%



③ 10%



Q) An automatic machine is designed to pack exactly 2.0 kg exactly, a sample of 100 tin was examine to test the machine the avg. weight was found to be 1.94 kg with standard deviation 0.10 kg. Is the machine working properly or not?

Solⁿ SE (Standard Error of mean) $= \dfrac{S}{\sqrt{n}}$

$\dfrac{0.10}{\sqrt{100}} = \dfrac{0.10}{10} = 0.01$

$Z = \dfrac{\bar{X} - \mu}{\sigma} = \dfrac{1.94 - 2}{.01} = \dfrac{-0.06}{0.01} = 6$

Since the computated value of z is more than the table value, we reject H_0 and conclude that the machine is not working properly

5% level = Z = 1.96

Ques) A company produce car tyre of avg life 40,000 km with standard deviation 3000 km. A sample of 64 tyre has been taken whose mean life is found 41,200 km test the significance diff at 5% level of significance.

Solⁿ $\bar{X} = 41{,}200$, $\mu = 40{,}000$, $\sigma = 3000$

$n = 64$

$\Rightarrow Z = \dfrac{41200 - 40000}{\frac{3000}{8}} \Rightarrow \dfrac{1200}{375} \Rightarrow 3.2$

5% level = 1.96

3.2 > 1.96

H_1 is true

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Comparision b/w two population mean

$$z = \frac{\bar{x}_1 - \bar{x}_2}{\sqrt{\frac{\sigma_1^2}{n_1} + \frac{\sigma_2^2}{n_2}}}$$

Q) A sample of 400 ladies has been taken from bhopal their avg purchase/week RS 5000. Another sample of 200 ladies has been taken from indore whose avg purchase/week is RS 5100 test the significance diff. at 5% level when it is given that σ is 3.5 and 4.5 for bhopal and indore respective

Sol^n $n_1 = 400$ $\bar{x}_1 = 5000$, $\sigma_1 = 3.5$
$n_2 = 200$ $\bar{x}_2$ 5100, $\sigma_2 = 4.5$

$$z = \frac{5000 - 5100}{\sqrt{\frac{(3.5)^2}{400} + \frac{(4.5)^2}{200}}}$$

$$\Rightarrow z = \frac{-100}{\frac{3.5}{20} + \frac{4.5}{14.14}} \qquad \Rightarrow z = \frac{-100}{\frac{49.49 + 90}{282.8}}$$

$\Rightarrow z = -100$

$$z = \frac{-100}{\sqrt{\frac{12.25}{400} + \frac{20.25}{200}}} \qquad \Rightarrow z = \frac{-100}{\sqrt{\frac{2450 + 8,100}{80,000}}}$$

$$\Rightarrow z = \frac{-100}{\sqrt{\frac{10,550}{80,000}}} \qquad \Rightarrow z = \frac{-100}{\sqrt{0.131}}$$

$$\Rightarrow z = \left|\frac{-100}{0.361}\right| \qquad \Rightarrow z = 277$$

5% level = 1.96
$\Rightarrow 277 > 1.96$
H_1 is True

Q $n_1 = 1000$, $n_2 = 32,000$, $\bar{x}_1 = 67.5$, $\bar{x}_2 = 6$
$\sigma_1 = \sigma_2 = 2.5$

Sol^n
$$z = \frac{67.5 - 68}{\sqrt{\frac{(2.5)^2}{1000} + \frac{(2.5)^2}{32,000}}} = \frac{-0.5}{\sqrt{\frac{6.25}{1000} + \frac{6.25}{32,00}}}$$

$$\Rightarrow z = \frac{-0.5}{\sqrt{\frac{200000 + 6250}{32,000,000}}} = \frac{-0.5}{\sqrt{\frac{206,250}{32,000,0}}}$$

$$\Rightarrow z = \frac{-0.5}{\sqrt{0.0064}} \qquad \Rightarrow z = \left|\frac{-0.5}{0.08}\right|$$

$\Rightarrow z = 6.25$

5% level = 1.96
$\Rightarrow 6.25 > 1.96$

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Q) In order to test whether the avg weekly maintenance cost of a fleet of buses is more than Rs. 500 a random sample of 49 buses was taken. The mean and the standard deviation were found to be Rs 506 and Rs. 42 Assume $\alpha = 0.025$

Sol^n $x = 500$, $n = 49$, $\mu = 506$, $\sigma = 42$

$$z = \left| \frac{x - \mu}{\frac{\sigma}{\sqrt{n}}} \right| \Rightarrow z = \left| \frac{500 - 506}{\frac{42}{\sqrt{49}}} \right|$$

$$\Rightarrow z = \left| \frac{-6}{\frac{42}{7}} \right| \Rightarrow z = \left| \frac{-6}{6} \right|$$

$$\Rightarrow \boxed{z = 1}$$

5% level = 1.96

$1 < 1.96$

H_1 is false

H_0 is true

Case III :- Practical steps involved in Test for proportion of succes

Compare b/w sample proportion & ~~pollution~~ population property

$$z = \frac{p - P}{\sqrt{P\left(\frac{1-P}{n}\right)}} = \frac{p - P}{\sqrt{\frac{Pq}{n}}}$$

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$$\Rightarrow SE = \sqrt{\frac{Pq}{n}} = \sqrt{\frac{(1-P)P}{n}}$$

$$\Rightarrow z = \frac{p - P}{SE}$$

$p =$ sample proportion

$P =$ Population proportion

$n =$ sample size

Q) A production manager claims that only 4% of the goods produced are defective. In a random sample of a batch of 600 units, 36 unit are found to be defective. Test the claim of production manager (a) at 5% level of significance and (b) at 1% level

Sol^n ~~p = 600~~ $p = \frac{36}{600}$ $P = 0.04$

$$SE = \sqrt{\frac{0.04 \times 0.96}{600}} \Rightarrow SE = 0.08$$

$$\Rightarrow z = 2.5$$

At 5% level :- $2.5 > 1.96$ H_1 is accepted & H_0 is rejected

At 1% level :- H_1 is rejected

H_0 is accepted

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Q) A coin is tossed 500 times and head is obtaind 251 times check the significance difference at 5% level.

Soln $P = 50\%$, $p = \frac{251}{500} = 0.502$

$n = 500$

$\Rightarrow SE = \sqrt{\frac{(1-P)P}{n}} = \sqrt{\frac{(1-0.5)\times 0.5}{500}}$

$\Rightarrow SE = \sqrt{\frac{0.25}{500}}$ $\Rightarrow SE = 0.022$

$\Rightarrow z = \frac{p-P}{SE} = \frac{0.5-0.5}{0.022} = 0$

at 5% level

$0 < 1.96$

Ho is accepted

Q) sky packets guarantee 90 percent of their deliveries are on time. In a recent week 81 deliveries were made of which 6 were late. sky packets Managing Director says with 95% confidence that there has been a significant improvement in deliveries. Should the Managing director's statement be accepted?

Soln $P = 0.9$, $p = \frac{6}{81} = 0.07$, $n = 81$

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$SE = \sqrt{\frac{P(1-P)}{n}} = \sqrt{\frac{0.9(1-0.07)}{81}} = \sqrt{\frac{0.837}{81}}$

$SE = \sqrt{0.010} = SE = 0.01$

$z = \left|\frac{p-P}{SE}\right| = \left|\frac{0.07-0.9}{0.01}\right| = \left|\frac{-0.83}{0.01}\right|$

$z = 8.3$

Case - IV - Practical steps involved in the test for diff. b/w population properties

$z = \frac{P_1 - P_2}{SE_s}$ $SE_{(P_1-P_2)} = \sqrt{Pq\left(\frac{1}{n_1}+\frac{1}{n_2}\right)}$

$P = \frac{n_1P_1 + n_2P_2}{n_1+n_2}$ or $\frac{x_1+x_2}{n_1+n_2}$

Where, x_1, x_2 stands for the no. of occurance in the two sample of size of n_1 & n_2 respectively P_1 and P_2 = proportion of succes in sample 1 is P_1 and proportion of sucess in sample 2 is P_2

Q) In a large city A, 20% of a random sample of 1000 school children had defective eye sight. In another large city, B, 15% of a random sample of 2000 children had the same defect. Is this diff. b/w the two proportion significant? Obtain 95% confidence limits for the difference in the population proportion

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Soln A :- B :-

$P_1 = 20\%$ $P_2 = 15\%$

$n_1 = 1000$ $n_2 = 2000$

$$P = \frac{n_1 P_1 + n_2 P_2}{n_1 + n_2} = \frac{1000 \times \frac{20}{100} + 2000 \times \frac{15}{100}}{1000 + 2000}$$

$$P = \frac{200 + 300}{300} = \frac{500}{3000} = \frac{1}{6} = 0.167$$

$\boxed{P = 0.167}$

$$SE = \sqrt{Pq\left(\frac{1}{n_1} + \frac{1}{n_2}\right)} = 0.1503\left(\frac{1}{1000} + \frac{1}{2000}\right)$$

$$SE = \sqrt{0.1503 \times \left(\frac{2000 + 1000}{2000000}\right)}$$

$$\Rightarrow SE = \sqrt{0.15 \times \frac{3000}{2000000}} = \sqrt{0.15 \times 0.0015}$$

$\Rightarrow \boxed{SE = 0.0144}$

$$\Rightarrow Z = \frac{\cancel{0.20} - \cancel{0.15}}{\cancel{0}} \quad \frac{P_1 - P_2}{SE} = \frac{0.20 - 0.15}{0.0144}$$

$\Rightarrow Z = 3.47211$

at 5% level

$3.472 > 1.96$

H_1 is accepted

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Estimation

\# Statistical Inference :- The process of drawing inference about population on the basis of sample data is called statistical inference.

⇒ Estimation ⇒ Testing of hypothesis

The process in which we obtained the value of unknown population with the help of sample data

⇒ Estimate :- An estimate is the numeric value of the estimation

Types of estimation:-

1. Point estimation 2. Interval estimation

★ Estimator :- It is a rule, formula or function that tells how to calculate an estimate

1. Point estimation :- When an estimate for the unknown population parameter is express by the single value is called point estimate

2. Interval estimation :- When an estimate for the unknown population parameter is expressed by a range of values within which the population parameter is expected to occur is called interval estimation

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Q) A random sample of $n=6$ has an elements 6, 10, 13, 14, 18, 20 compute a point estimate of
① population mean
② population of SD
③ standard error of the mean

Sol ① Mean $\bar{x} = \frac{\Sigma x}{n} = \frac{6+10+13+14+18+20}{6}$

$= \frac{81}{6} = 13.5$//

② $S.D = \sqrt{\frac{\Sigma x^2}{n} - \left(\frac{\Sigma x}{n}\right)^2}$

$\Rightarrow S.D = \sqrt{\frac{1221}{6} - \left(\frac{81}{6}\right)^2} = \sqrt{\frac{7350-6561}{36}}$

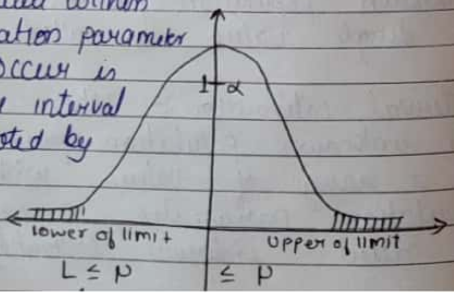
$= \sqrt{\frac{789}{36}} = \sqrt{21.91666}$

$\Rightarrow SD = 4.6815$ ans//

③ $S.E = \frac{SD}{\sqrt{n}} = \frac{4.68}{\sqrt{6}} = \frac{4.68}{2.44} = 1.91$ ans//

Confidence Interval

The range of value within which the population parameter expected to occur is called confidence interval and it is denoted by $1-\alpha$



$L \leq p \leq p$

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① Confidence I for p when $n \geq 30$ and σ know

$\bar{x} + z_{\alpha/2} \frac{\sigma}{\sqrt{n}}$

$\bar{x} \rightarrow$ sample mean $z_{\alpha/2}$ = critical value of z

n = sample size σ = S.D

② Confidence I for p when $n \geq 30$ and σ unknown

$\bar{x} \pm z_{\alpha/2} \frac{s}{\sqrt{n-1}}$, where, $s = \frac{s}{\sqrt{n-1}}$

③ Confidence I for p when $n < 30$ and σ known

$\bar{x} \pm z_{\alpha/2} \frac{\sigma}{\sqrt{n}}$

④ Confidence I for p when $n < 30$ and σ unknown

$\bar{x} \pm t_{\alpha/2} \frac{s}{\sqrt{n-1}}$

Critical value of t distribution

⑤ Confidence I for p when $n_1 \& n_2 \geq 30$ and σ_1^2 & σ_2^2 known

$(\bar{x}_1 - \bar{x}_2) + z_{\alpha/2} \sqrt{\frac{\sigma_1^2}{n_1} + \frac{\sigma_2^2}{n_2}}$

⑥ Confidence I for p when $n_1 \& n_2 < 30$ and σ_1^2 & σ_2^2 known

$(\bar{x}_1 - \bar{x}_2) + z_{\alpha/2} \sqrt{\frac{\sigma_1^2}{n_1} + \frac{\sigma_2^2}{n_2}}$

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⑦ Confidence I for μ when n_1 & $n_2 \geq 30$ & σ_1^2 & σ_2^2 unknown

$$(\bar{x}_1 - \bar{x}_2) \pm z_{\alpha/2} \sqrt{\frac{s_1^2}{n_1} + \frac{s_2^2}{n_2}}$$

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Q) find Confidence Interval for the population mean at 95% Confidence level in the following case

Case	Population Size	Sample Size	Population S.D	Sample S.D	Sample mean
(a)	Not available	36	3	4	100
(b)	Not available	36	–	4	100
(c)	600	36	3	4	100
(d)	600	36	–	4	100
(e)	720	36	3	4	100
(f)	720	36	–	4	100
(g)	900	36	3	4	100
(h)	900	36	–	4	100

Solⁿ level of significance = 100 − 95 ≥ 1.96
= 5%.

ⓐ Confidence level = 95%, Confidence z (1.96)

$$SE = \frac{\sigma}{\sqrt{n}} = \frac{3}{\sqrt{36}} = \frac{3}{6} = \frac{1}{2} = 0.5$$

$$\Rightarrow \bar{x} \pm z\,SE \Rightarrow \frac{100 \pm (1.96) \times 0.5}{\sqrt{n-1}}$$

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$\Rightarrow$ upper condition :- $\frac{100 + (1.96) \times 0.5}{\cancel{\sqrt{35}}}$

100.98

$\Rightarrow$ lower confidence :- $100 - (1.96) \times 0.5$

ⓑ $z = 1.96$ $\Rightarrow$ sample s.D

$$SE = \frac{S}{\sqrt{n-1}} \quad \frac{4}{\sqrt{35}} = 0.676$$

$$\Rightarrow \bar{x} \pm z\,SE \Rightarrow 100 \pm (1.96) \times 0.67$$

Lower Confidence :- $100 - (1.96) \times 0.67$
98.68

Upper Confidence :- $100 + (1.96) \times 0.67$
101.31

ⓒ $z = 1.96$; if sample is given

sampling fraction $= \frac{n}{N} = \frac{\cancel{36}\,6}{\cancel{600}\,100}$
$= 0.06$

NOTE if SE is greater than sampling fraction then do this :-

$$SE = \frac{\sigma}{\sqrt{n}} \times \sqrt{\frac{N-n}{N-1}}$$

$$SE = \frac{3}{\sqrt{36}} \times \sqrt{\frac{600-36}{600-1}} = \frac{\cancel{36}}{\cancel{62}} \times \sqrt{\frac{564}{599}}$$

$SE = 0.5 \times \sqrt{0.941} = 0.5 \times 0.97$

$SE = 0.485$

$\bar{x} \pm z\,SE = 100 + (1.96)\,0.485$

Upper $= 100 + (1.96)0.485$
$= 100.95$

lower $= 100 - (1.96)0.485$
$= 99.04$

(d) $z = 1.96$ Sampling fraction $= \frac{36}{600} = 0.06$

$SE = \frac{s}{\sqrt{n-1}} = \frac{4}{\sqrt{35}} = 0.67$

$SE = \frac{\sigma}{\sqrt{n}} \times \sqrt{\frac{N-n}{N-1}} = \frac{4}{\sqrt{35}} \times \sqrt{\frac{600-36}{600-1}}$
$= 0.67 \times 0.97$
$SE = 0.64$

$\bar{x} \pm z\,SE = 100 \pm (1.96)(0.64)$

upper $= 100 + 1.96 \times 0.64$
101.25

lower $= 100 - 1.96 \times 0.64$
98.74

$z = 1.96$, sampling function :- $\frac{36}{720} = 0.05$

$SE = \frac{\sigma}{\sqrt{n}} \times \sqrt{\frac{N-n}{N-1}} = \frac{3}{\sqrt{36}} \times \sqrt{\frac{720-36}{720-1}}$

$SE = \frac{3}{6} \times \sqrt{\frac{684}{719}} = 0.5 \times \sqrt{0.95}$

$SE = 0.5 \times 0.97 = 0.485$

$\bar{x} \pm z\,SE = 100 \pm (1.96)\,0.485$

Upper $= 100 + 1.96 \times 0.485$
100.95

lower $= 100 - 1.96 \times 0.485$
99.04

(f) $Z = 1.96$, $N = 720$, $n = 36$
$SF = \frac{36}{720} = 0.05 < 0.5$

$SE = \frac{4}{\sqrt{36}} \times \sqrt{\frac{720-36}{720-1}} = 0.66 \times 0.97$

$SE = 0.65$

$\bar{x} \pm z\,SE = 100 \pm (1.96)\,0.65$

upper $= 100 + 1.96 \times 0.65$
101.27

lower $= 100 - 1.96 \times 0.65$
98.72

(g) $Z = 1.96$, $N = 900$, $n = 36$
$SF = \frac{36}{900} = 0.04$

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$SE = \frac{3}{\sqrt{36}} \times \sqrt{\frac{900-36}{900-1}} = \frac{8}{62} \times \sqrt{\frac{864}{899}}$

$SE = 0.5 \times \sqrt{0.96} = 0.5 \times 0.97 = 0.485$

$\bar{x} \pm zSE = 100 \pm 1.96 \times 0.485$

Upper $= 100 + 1.96 \times 0.48$
100.95

lower $= 100 - 1.96 \times 0.485$
99.04

(h) $z = 1.96$ $N = 900$, $n = 36$

$SF = \frac{n}{N} = \frac{36}{900} = 0.04$

$SE = \frac{4}{\sqrt{36}} \times \sqrt{\frac{900-36}{900-1}} = 0.66 \times 0.97$

$SE = 0.64$

$\bar{x} \pm zSE = 100 \pm 1.96 \times 0.64$

upper $= 100 + 1.96 \times 0.64$
101.25

lower $= 100 - 1.96 \times 0.64$
98.74

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Q) find Confidence Interval for the population mean at 95 confidence level in the following cases:- Assume that the population mean

case	Population size	sample size	Population SD	Sample SD	Sample mean
(a)	Not available	25	3	4	100
(b)	400	25	3	4	100
(c)	500	25	3	4	100
(d)	600	25	3	4	100

Soln (a) confidence level = 95 %, $z = 1.96$
$n = 25$

$SE = \frac{\sigma}{\sqrt{n}} = \frac{3}{\sqrt{25}} = \frac{3}{5} = 0.6$

$\Rightarrow \bar{x} \pm zSE \Rightarrow 100 \pm 1.96 \times 0.6$

$\Rightarrow$ upper $= 100 + 1.96 \times 0.6$
101.17

$\Rightarrow$ lower $= 100 - (1.96) \times 0.6$
98.82

(b) $z = 1.96$, sampling fraction $= \frac{25}{400}$
$N = 400$, $n = 25$

$0.06 < 0.5$

$SE = \frac{\sigma}{\sqrt{n}} \times \sqrt{\frac{N-n}{N-1}} = \frac{4}{5} \times \sqrt{\frac{400-2}{400-}}$

$SE = 0.8 \times \sqrt{\frac{375}{399}} = 0.8 \times 0.98$

$SE = 0.\cancel{76}\,57$

$\bar{x} \pm ZSE = 100 \pm (1.96) \times (0.74)$

$\Rightarrow$ upper $= 100 + 1.96 \times 0.76$
101.48

$\Rightarrow$ lower $= 100 - (1.96) \times 0.76$
98.51

(c) $N = 500$, $n = 25$, $Z = 1.96$

$SF = \frac{25}{500} = 0.05$

$SE = \frac{\sigma}{\sqrt{n}} \sqrt{\frac{N-n}{N-1}} = \frac{3}{5} \times \sqrt{\frac{500-25}{500-1}}$

$= 0.6 \times \sqrt{\frac{475}{499}}$

$= 0.6 \times \sqrt{0.95}$

$= 0.6 \times 0.97$

$SE = 0.582$

$100 \pm 1.96 \times 0.58$

$\Rightarrow$ upper $= 100 + 1.96 \times 0.58$
101.13

lower $= 100 - 1.96 \times 0.58$
98.86

(d) $N = 600$, $n = 25$, $\sigma = 3$, $Z = 1.96$

$SF = \frac{25}{600} = 0.04$

$SE = \frac{\sigma}{\sqrt{n}} \times \sqrt{\frac{N-n}{N-1}} = \frac{3}{5} \sqrt{\frac{600-25}{600-1}}$

$SE = 0.6$

$\bar{x} \pm ZSE = 100 \pm 1.96 \times 0.6$

$\Rightarrow$ upper $= 100 + 1.96 \times 0.6$
101.17

$\Rightarrow$ lower $= 100 - 1.96 \times 0.6$
98.82

Q) find Confidence Interval for the population mean at 95% confidence level in the following cases:-

case	population size	sample size	sample SD	mean sample
(a)	Not available	25	4	100
(b)	400	25	4	100
(c)	500	25	4	100
(d)	600	25	4	100

(a) $Z = 1.96$

$SE = \frac{\sigma}{\sqrt{n-1}} = \frac{4}{5} = \frac{4}{4.8} = 0.83$

$\bar{x} \pm ZSE = 100 \pm 1.96 \times 0.8$

upper $= 100 + 1.96 \times 0.8$
101.562

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lower = $100 - 1.96 \times 0.8$
98.87

(b) $SE = \frac{n}{N} = \frac{25}{400} = 0.06$

$N = 400, \quad n = 25$

$SE = \frac{4}{5} \times \sqrt{\frac{400-25}{400-1}} = 0.8\sqrt{\frac{375}{399}}$

$SE = 0.8 \times \sqrt{0.93} = 0.8 \times 0.96$

$SE = 0.76$

$t = 2.06$

$\bar{X} \pm tSE = 100 \pm 2.06 \times 0.76$

$\Rightarrow$ upper = $100 + 2.06 \times 0.74$
101.56

$\Rightarrow$ lower = $100 - 2.06 \times 0.74$
98.47

(c) $N = 500, n = 25, S = 4$

$SE = \frac{25}{500} = 0.05$

$SE = \frac{4}{5} \times \sqrt{\frac{500-25}{500-1}} = 0.8 \times 0.97$

$SE = 0.77$

$\bar{X} \pm tSE = 100 \pm 2.06 \times 0.77$

$\Rightarrow$ upper = $100 + 2.06 \times 0.77$
101.58

$\Rightarrow$ lower = $100 - 2.06 \times 0.77$
98.41

(d) $N = 600, \quad n = 25, \quad S = 4$

$SE = \frac{25}{600} = 0.04$

$SE = \frac{4}{\sqrt{25-1}} = \frac{4}{4.8} = 0.81$

$\bar{X} \pm tSE = 100 \pm 2.06 \times 0.81$

$\Rightarrow$ upper = $100 + 2.06 \times 0.81$
101.66

$\Rightarrow$ lower = $100 - 2.06 \times 0.81$
98.33

Population Proportion

$\Rightarrow$ Practical steps involved in the construction of confidence involved estimate of the population proportion

1. When population proportion is known

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if $SF > 0.05$

$$SE_s = \sqrt{\frac{PQ}{n}} \qquad SE_p = \sqrt{\frac{PQ}{n}} \times \sqrt{\frac{N-n}{N-1}}$$

P = population proportion
Q = 1-P
n = sample size

2. When population proportion is not known

$$SE_s = \sqrt{\frac{PQ}{n}}, \quad SE_p = \sqrt{\frac{pq}{n}} \times \sqrt{\frac{N-n}{N-1}}$$

Q) find Confidence Interval for the population proportion at 95% Confidence level in the following cases:

case	Batch size	Defective in Batch	Sample size	Defective in Sample
a	—	4%	600	36
b	—	—	600	36
c	7500	300	600	36
d	7500	—	600	36
e	12000	480	600	36
f	12000	—	600	36
g	20000	800	600	36
h	20000	—	600	36

Soln $P = 0.04$, $q = 1 - 0.04 = 0.6$, $n = 600$

$$SE = \sqrt{\frac{pq}{n}} = \sqrt{\frac{0.04 \times 0.96}{600}} = \sqrt{\frac{0.000}{4}}$$

$$SE = \sqrt{\frac{0.0384}{600}} = \sqrt{0.00064} = 0.008$$

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$P \pm ZSE \Rightarrow P = \frac{36}{600} = \cancel{0.06} = 0.06$

$0.04 \pm 1.96 \times 0.08$

$\Rightarrow$ upper $= 0.04 + 1.96 \times 0.08$
$\cancel{0.196}$ 0.075

$\Rightarrow$ lower $= 0.04 - 1.96 \times 0.008$
0.0443

(b) $P = \frac{36}{600} = 0.06$, $q = 1 - 0.06 = 0.94$

$$SE = \sqrt{\frac{0.06 \times 0.94}{36}} \times \sqrt{\frac{600-36}{600-1}}$$

$$SE = \sqrt{\frac{0.0564}{36}} \times \sqrt{\frac{564}{599}} = \sqrt{0.0015} \times \sqrt{0.94}$$

$SE = 0.039 \times 0.97 = 0.036$

$P \pm ZSE = \begin{matrix} P + ZSE \\ P - ZSE \end{matrix}$

$\Rightarrow P + ZSE = 0.06 + 1.96 \times 0.036$

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(c) $sf \not{p} = \frac{n}{N} = \frac{600}{7500} = 0.08 \geq 0.05$

$P = \frac{300}{7500} = 0.04$

$Q = 1 - P = 1 - 0.04 = 0.96$

$n = 600$

$SE_p = \sqrt{\frac{pq}{n}} \times \sqrt{\frac{N-n}{N-1}} = \sqrt{\frac{0.04 \times 0.96}{600}} \times \sqrt{\frac{7500-600}{600-1}}$

$SE = \sqrt{0.000064} \times \sqrt{11.51}$

$SE = 0.008 \times 3.39$

Q. a) $q = 1 - 0.04$

$q = 0.96$

$p = 0.04$

$SE = \sqrt{\frac{PQ}{n}} = \sqrt{\frac{(0.04)(0.96)}{600}} = \sqrt{\frac{0.0384}{600}} = \sqrt{0.000064}$

$SE = 0.008$

$P \pm Z(SE)$

$P \pm (1.96)(0.008)$

$P \pm (0.01568)$

for $P \Rightarrow P = 36/600 \Rightarrow P = 0.06$

$0.06 \pm (0.01568)$

upper 0.07568

lower 0.04432

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b) $P = 36/600 = 0.06$

$Q = 1 - 0.06 = 0.94$

$SE = \sqrt{\frac{PQ}{n}} = \sqrt{\frac{(0.06)(0.94)}{600}} = \sqrt{\frac{0.0564}{600}} = \sqrt{0.000094}$

$SE = 0.009695$

$P \pm Z(SE)$

$0.06 \pm (1.96)(0.009695)$

0.06 ± 0.0190022

upper 0.0790022

lower 0.0409978

c) $\frac{300}{7500} \times 100^{4} = 4\%$

{to check which formula to apply}

$N = 7500$

$P = 0.04$

$Q = 0.96$

$n = 600$

$P = \frac{600}{7500} = 0.08$

$SE = \sqrt{\frac{PQ}{n}} \times \sqrt{\frac{N-n}{N-1}}$

$= \sqrt{\frac{(0.04)(0.96)}{600}} \times \sqrt{\frac{7500-600}{7500-1}}$

$= 0.008 \times 0.95921$

$= 0.00767368$

$P \pm Z(SE) = (0.04) \pm (1.96)(0.00767368)$

$= (0.04) \pm 0.0150$

upper $\Rightarrow$ 0.055

lower $\Rightarrow$ 0.02496

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d) $\frac{600}{7500} = 0.08$

for $SE = \sqrt{\frac{PQ}{n}} \times \sqrt{\frac{N-n}{N-1}}$

for $P \Rightarrow \frac{36}{600} = 0.06$

$Q = 0.94$

$N = 7500$

$n = 600$

$SE = \sqrt{\frac{(0.06)(0.94)}{600}} \times \sqrt{\frac{7500-600}{7500-1}}$

$SE = 0.009695 \times 0.95921$

$SE = 0.009299$

$P \pm Z(SE)$

$P \pm (1.96)(0.009299)$

0.06 ± 0.01822

lower 0.04178

upper 0.07822

e $SF = \frac{n}{N} = \frac{600}{12000} = 0.05 \Rightarrow 0.05$

$P = \frac{480}{12000} = 0.04$, $Q = 1 - 0.04 = 0.96$

$n = 600$, $N = 1200$

$SE = \sqrt{\frac{PQ}{n}} \times \sqrt{\frac{N-n}{N-1}} = \sqrt{\frac{0.04 \times 0.96}{600}} \times \sqrt{\frac{12000-600}{12000-1}}$

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$SE = \sqrt{\frac{0.24}{600}} \times \sqrt{\frac{600}{11,999}} = \sqrt{0.0004} \times \sqrt{0.30}$

$SE = 0.02 \times 0.54 = 0.0108$

$P \pm ZSE$:- $P = \frac{36}{600} = 0.06$

upper :- $P + ZSE = 0.06 + 1.96 \times 0.0108$

$= 0.081$

lower :- $P - (Z)SE = 0.06 + 1.96 \times 0.0108$

$= 0.038$

$SE = \sqrt{\frac{0.0384}{600}} \times \sqrt{\frac{11,400}{11,999}} = \sqrt{0.000064} \times \sqrt{0.950}$

$SE = 0.008 \times 0.97 = 0.00776$

$P \pm ZSE \Rightarrow P = \frac{36}{600} = 0.06$

upper :- $p + ZSE = 0.06 + 1.96 \times 0.00776$

$= 0.0752$

lower :- $p - ZSE = 0.06 - 1.96 \times 0.00776$

0.0447

(f) $SF = \frac{n}{N} = \frac{600}{12,000} = 0.05 = 0.05$

$P = \frac{36}{600} = 0.06$, $Q = 1 - P = 0.96$

$n = 600$, $N = 12,000$

$SE = \sqrt{\frac{PQ}{n}} \times \sqrt{\frac{N-n}{N-1}} = \sqrt{\frac{0.06 \times 0.96}{600}} \times \sqrt{\frac{12000-600}{12000-1}}$

$= \sqrt{\frac{0.576}{600}} \times \sqrt{\frac{11,400}{11,999}} = \sqrt{0.00096} \times \sqrt{0.95}$

$\Rightarrow SE = 0.030 \times 0.97 = 0.0291$

$P \pm z\,SE \quad = \quad P = \frac{36}{600} = 0.06$

upper $= 0.06 + 1.96 \times 0.0291$

$= \cancel{0.680}\ 0.117$

lower $= 0.06 - 1.96 \times 0.291$

$\cancel{0.51}\ 0.0296$

(g) $N = 20000$, $SF = \frac{600}{20000} = 0.03 < 0.05$

$P = \frac{800}{20000} = 0.04$

$Q = 0.96$

$n = 600$

$SE = \sqrt{\frac{PQ}{n}} = \sqrt{\frac{0.04 \times 0.96}{600}} = \sqrt{\frac{0.0384}{600}}$

$SE = \sqrt{0.000064} = 0.008$

$P \pm z\,SE \Rightarrow P = \frac{36}{600} = 0.06$

upper $= 0.06 + 1.96 \times 0.008$

0.07568

lower $= 0.06 - 1.96 \times 0.008$

0.04432

(h) $SF = \frac{600}{20000} = 0.03 < 0.05$

$P = \frac{36}{600} = 0.06$

$q = 0.94$, $n = 600$

$SE = \sqrt{\frac{Pq}{n}} = \sqrt{\frac{0.06 \times 0.94}{600}}$

$SE = 0.0096$

$P \pm z\,SE \Rightarrow P = \frac{36}{600} = 0.06$

upper $= 0.06 + 1.96 \times 0.0096$

0.0788

lower $= 0.06 - 1.96 \times 0.0096$

0.0411

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Test

Parametric Test
- → T- test
- → z-test
- → f- test

Non-Parametric Test
- → chi test
- → chi^2 test

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⇒ T- test

for one tail

$$t = \frac{\bar{x} - \mu}{s \times \sqrt{n}} , \quad s = \sqrt{\frac{\Sigma(x-\bar{x})}{n-1}} \text{ or}$$

for 2 tail

$$\sqrt{\frac{\Sigma d^2 - n(\bar{d})^2}{n-1}}$$

$$\bar{x} = \frac{\Sigma x}{n} , \quad \bar{d} = \frac{\Sigma d}{n}$$

$\bar{d}$ = deviation
$\bar{x}$ = sample mean
μ = population mean
s = sample SD

☆ Uses / application of T- test

- Size of sample should be small ($n<30$)
- Degree of freedom is $v = n-1$
- T test is used for test of significance of regration coefficient in regelation model
- We use the statics when parameter of population are normal
- when population variable are unknown
- Co Relation of coefficient in population is 0.

Q) A fertiliser mixing machine is set to give 4 kg of nitrate for every quintal bag of fertilizers. Five 100 kgs bags are examined. The percentage of nitrate are : 2, 6, 4, 3, 1. Is

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there reason to believe that the machine is defective

Sol^n

X	$d = x-2$	d^2
2	0	0
6	4	16
4	2	4
3	1	1
1	-1	1
$\Sigma x = 16$	$\Sigma d = 6$	$\Sigma d^2 = 22$

$$\bar{X} = \frac{\Sigma X}{n} = \frac{16}{5} = 3.2$$

$$\bar{d} = \frac{\Sigma d}{n} = \frac{6}{5} = 1.2$$

$$s = \sqrt{\frac{\Sigma d^2 - n(\bar{d})^2}{n-1}} = \sqrt{\frac{22 - 5(1.2)^2}{4}}$$

$$s = \sqrt{\frac{22 - 5(1.44)}{4}} \Rightarrow s = 1.92$$

$$t = \frac{\bar{X} - \mu}{s \times \sqrt{n}} = \frac{3.2 - 4}{1.92 \times 2.23} = \frac{-0.8}{4.28}$$

$$\boxed{t = 0.18}$$

degree of freedom

$v = n-1 = 5-1 = 4$

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Value at 4 in two tail = 2.776

0.18 < 2.776

Ho is accepted

Difference b/w the means of two independent Random Samples

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$$t = \frac{|\bar{X}_1 - \bar{X}_2|}{S} \times \sqrt{\frac{n_1 n_2}{n_1 + n_2}}$$

$$\bar{X}_1 = \frac{\Sigma X_1}{n_1}, \quad \bar{X}_2 = \frac{\Sigma X_2}{n_2}$$

(a) where deviation are taken from actual mean

$$S = \sqrt{\frac{\Sigma (X_1 - \bar{X}_1)^2 + \Sigma (X_2 - \bar{X}_2)^2}{n_1 + n_2 - 2}}$$

(b) where deviations are taken from assumed mean

$$S = \sqrt{\frac{\Sigma (X_1 - A_1)^2 + \Sigma (X_2 - A_2)^2 - n_1 (\bar{X} - A_1)^2 - n_2 - (\bar{X}_2 - A_2)^2}{n_1 + n_2 - 2}}$$

A_1 & A_2 = Assumed mean of first sample & 2nd sample respectively

$\bar{X}_1$ & $\bar{X}_2$ = Actual mean of first sample & 2nd sample respectively

(c) Where the individual standard deviation of both the samples are given

$$S = \sqrt{\frac{(n_1 - 1) s_1^2 + (n_2 - 1) s_2^2}{n_1 + n_2 - 2}}$$

$$t = \frac{|\bar{X}_1 - \bar{X}_2|}{S} \times \sqrt{\frac{n_1 n_2}{n_1 + n_2}}$$

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8) A group of 5 patient treated with medicine A weigh 42, 39, 48, 60, 41 kg. A second group of 5 patient treated with medicine B weigh 38, 42, 48, 67, 40 kg. Do the two medicines differ significantly with regard to their effect in increasing weight?

Degree of freedom	5	8	9	10
Value of t at 5% level	2.57	2.31	2.26	2.23

Soln

X_1	$X_1 - \bar{X}_1 = X_1 - 46$	$(X_1 - \bar{X}_1)^2$	X_2	$X_2 - \bar{X}_2 = X_2 - 47$	$(X_2 - \bar{X}_2)^2$
42	-4	16	38	-9	81
39	-7	49	42	-5	25
48	2	4	48	1	1
60	14	196	67	20	400
41	-5	25	40	-7	49
Σ46		Σ290	Σ47		Σ556

Soln

$$S = \sqrt{\frac{\Sigma (X_1 - \bar{X}_1)^2 + \Sigma (X_2 - \bar{X}_2)^2}{n_1 + n_2 - 2}}$$

$$\Rightarrow S = \sqrt{\frac{290 + 556}{5 + 5 - 2}} = \sqrt{\frac{846}{8}} \quad \sqrt{105.75}$$

$$\Rightarrow S = 10.283$$

$$t = \frac{|\bar{X}_1 - \bar{X}_2|}{S} \times \sqrt{\frac{n_1 n_2}{n_1 + n_2}}$$

$$t = \frac{46 - 47}{10.283} \times \sqrt{\frac{5 \times 5}{10}}$$

$$t = 0.097 \times \sqrt{2.5} = 0.097 \times 1.58$$

$t = 0.153$

$v = n_1 + n_2 - 2 = 5 + 5 - 2 = 8$

$t_{2.306} > t_{0.153}$

Ho is accepted

Q) Two types of batteries are tested for their length of life and the following data are obtained

	No. of Sample	Mean life in hours	Variance
Type. A	9	600	121
Type. B	8	640	144

Is there significant diff b/w the means of the two batteries at 5% level of significance? Given the following

Degree of freedom	15	16	17
Value of t at 5% level	2.13	2.12	2.11

Soln

$$s = \sqrt{\frac{(n_1-1)s_1^2 + (n_2-1)s_2^2}{n_1+n_2-2}}$$

$$s = \sqrt{\frac{(9-1)(121)^2 + (8-1)144^2}{9+8-2}}$$

$$s = \sqrt{\frac{8 \times 121 \times 121 + 7 \times 144 \times 144}{15}}$$

$$s = \sqrt{\frac{262,280}{15}} = \sqrt{17,485.33}$$

$$S = 132.23$$

$$t = \left|\frac{\bar{x}_1 - \bar{x}_2}{s}\right| \times \sqrt{\frac{n_1 n_2}{n_1+n_2}}$$

$$t = \left|\frac{600 - 640}{132.23}\right| \times \sqrt{\frac{9 \times 8}{9+8}}$$

$$t = \left|\frac{-40}{132.23}\right| \times \sqrt{\frac{72}{17}}$$

$t = 0.302 \times \sqrt{4.23}$

$t = 0.302 \times 2.056$

$t = 0.6209$

degree of freedom :- $v = n_1 + n_2 - 2 = 9 + 8 - 2 = 15$

$t_{2.13} > t_{0.62}$

Ho is true

Diff. b/w two dependent sample

$$s = \sqrt{\frac{\Sigma d^2 - n(\bar{d})^2}{n-1}} \quad \text{or} \quad \sqrt{\frac{n\Sigma d^2 - (\Sigma d)^2}{N(n-1)}}$$

$$t = \frac{\bar{d} - 0}{s} \times \sqrt{n} \quad \text{or} \quad t = \frac{\bar{d}\sqrt{n}}{s}$$

$d = Y - X \qquad \bar{d} = \Sigma d / n$

Q1) A certain medicine was given to each of the 5 patient. The results are given below

	I	II	III	IV	V
Weight before medicine	42	39	48	60	41
Weight after medicine	38	48	48	67	40

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Test whether there is any change in weight after the medicine at 5% level of significance. Given the following

Degree of freedom	4	5	8	9	10
Value of t at 5% level	2.78	2.57	2.31	2.26	2.23

Sol^n

Before X	After Y	d = Y − X	d^2
42	38	−4	16
39	42	3	9
48	48	0	0
60	67	7	49
41	40	−1	1
		Σd 5	Σ 75

$\bar{d} = 1$

$$s = \sqrt{\frac{\Sigma d^2 - n(\bar{d})^2}{n-1}} = \sqrt{\frac{75 - 5}{4}} = \sqrt{\frac{70}{4}}$$

$$s = \sqrt{17.5} = 4.18$$

$$t = \frac{\bar{d} - 0}{s} \times \sqrt{n} = \frac{1-0}{s}\sqrt{5}$$

$$t = \frac{2.23}{5} = 0.446$$

$V = n - 1 = 5 - 1 = 4$

t => t

2.7 0.446

H0 accepted

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Chi Square Test

- Chi square test is introduced by Karl pearson
- Chi square test is sampling analysis for test significance of population variance
- Chi square is non parametric test. It can be used for test for Goodness of fit
- Chi - square Test is use simple random sampling method
- Chi - square Test value ~~max~~ lies b/w 0 to 1

Condition for using chi - square test

- Total frequency (Sample) > 50 n > 50
- samples are independent
- Cell frequency are linear
- Expected frequency will not be small if it is small e<5 then cooling frequency technique is used

$$\chi^2 = \Sigma\left(\frac{(O-E)^2}{E}\right)$$ O = Observed value

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E = Expected frequency

Q1) In a survey 200 girls of which 40% were intelligent, 30% had uneducated father, with 20% of unintelligent girls had educated father. Do these figure supports the hypothesis that educated father have intelligent girls? Test at 5% level of significance

Soln

	Intelligent girl	unintelligent girl	Row Total
Educated Father	56	24	80
UnEducated Father	24	96	120
coloumn total	80	120	200

O	E	O-E	$(O-E)^2/E$
56	32	24	576/32 = 18
24	48	-24	576/48 = 12
24	48	-24	576/48 = 12
96	72	24	576/72 = 8
		Σ	50

$E = \frac{80 \times 80}{200} = 32$, $\frac{80 \times 120}{200} = 48$, $\frac{120 \times 80}{200} = 48$

$\frac{120 \times 120}{200} = 72$

$\chi^2 = (\cancel{576}) \; 50$

$dF = n-1 = 4-1 = 3$

$50 > 7.815$

H_1 accepted

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Q) In a survey of 2000 students of which 55% were undergraduate 20% favoured the autonomous colleges while 40% of the post graduate opposed. Test at 5% level of significance that opinions of under-graduate and post graduate students on autonomous status of colleges are independent

Soln

	favourd	opposed	Row Total
UG	220	880	1100
PG	540	360	900
Coloumn total	760	1240	2000

O	E	O-E	$(O-E)^2$	
220	418	-198	39,204/418	= 93.78
880	682	198	39204/682	57.48
540	342	198	39204/342	114.63
360	558	-198	39204/557	70.25
				336.14

$E = \frac{1100 \times 760}{2000} = 418$, $\frac{1100 \times 1240}{2000} = 682$,

$\frac{900 \times 1240}{2000} = \cancel{760}$, $\frac{900 \times 760}{2000} = 342$

$\chi^2 = \Sigma \frac{(O-E)^2}{E} = 336.14$

$dF = n-1 = 4-1 = 3$

$336.14 > 7.815$

H_1 accepted

Sign Test

$$z = \frac{x - \mu}{\sigma}$$

Ques1) The nutritionist and medical doctors have always believed that vitamin C is highly effective in reducing the incidents of cold. To test this belief, a random sample of 13 person is selected and they are given large daily doses of vitamin C under medical supervision over a period of 1 year. The number of person who catch cold during the year is recorded and a comparision is made with the number of cold contacted by each such person during the previous year. This comparision is recorded as follows along with the sign of the change

Sol^n

Observation	Without Vitamin C	With vitamin C	+/-
1	7	2	+
2	5	1	+
3	2	0	+
4	3	1	+
5	8	3	+
6	2	2	0
7	4	3	+
8	4	5	−
9	3	1	+
10	7	4	+
11	6	4	+
12	2	3	−
13	10	4	+

earlier $n = 13$, now after neglecting 0 $n = 12$

$$z = \frac{x - \mu}{\sigma}, \quad \mu = np$$

$$\Rightarrow \mu = 12 \times 0.5 = 6$$

$$\Rightarrow \sigma = \sqrt{npq} = \sqrt{12 \times 0.5 \times 0.5} = \sqrt{3}$$

$$\sigma = 1.73$$

$$x = 9.5$$

$$\Rightarrow z = \frac{9.5 - 6}{1.73} = \frac{3.5}{1.73} = 2.023$$

NOTE:- x = all +ves − p so here x = ~~10~~ ~~we assume it to be~~ 9.5

$1.96 < 2.023$

Ho is rejected.

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Ques2) The median age of tourist who come to India is claimed to be 40 years. A random sample of 18 tourists gives the following ages :- 24, 18, 37, 51, 56, 38, 45, 45, 29, 48, 39, 26, 38, 43, 62, 30, 66, 41

Testing the hypothesis using $\alpha = 0.05$ level of significance

soln

Observation ages	mean age	+/-
24	40	-
18	40	-
37	40	-
51	40	+
56	40	+
38	40	-
45	40	+
45	40	+
29	40	-
48	40	+
39	40	-
26	40	-
38	40	-
43	40	+
62	40	+
30	40	-
66	40	+
41	40	+

$X = 9 = 8.5$, $n = 18$, $p = 0.5$

$\mu = np = 9$

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$\sigma = \sqrt{npq} = \sqrt{18 \times 0.5 \times 0.5}$ $\sqrt{4.5}$

$\sigma = 2.121$

$z = \dfrac{8.5 - 9}{2.12} = \dfrac{-0.5}{2.12}$

$\boxed{z = -0.235}$

$0.235 < 1.96$

H_0 accepted

Ques3) In a beauty contest there are two judges who have to rate 12 contestant. The rating have a score from 1 to 5. The score given by the judge are as follows :-

Contestant	Judge I	Judge II
1	2	3
2	1	2
3	4	2
4	4	3
5	3	4
6	3	2
7	4	2
8	2	1
9	4	3
10	1	1
11	3	3
12	3	5

soln

Contestant	Judge 1	Judge 2
1	2	3
2	1	2
3	4	2

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4	4	3
5	3	2
6	3	2
7	4	2
8	2	1
9	4	3
10	1	1
11	3	3
12	3	3

$\Rightarrow {}^9C_6 \ (0.5)^6 (0.5)^{9-6} = \frac{9!}{6!\ 3!} \times (0.5)^6 \times (0.5)^3$

$= 0.164$

$\Rightarrow {}^9C_7 \ (0.5)^7 (0.5)^2 = 0.0703$

$\Rightarrow {}^9C_8 \ (0.5)^8 (0.5)^1 = 0.0176$

$\Rightarrow {}^9C_9 \ (0.5)^9 (0.5)^0 = 0.00195$

$\Rightarrow {}^9C_6 \ (0.5)^6 (0.5)^3 + {}^9C_7 (0.5)^7 (0.5)^2 + {}^9C_8 (0.5)^8 (0.5)^1 + {}^9C_9 (0.5)^9 (0.5)^0$

$\Rightarrow 0.164 + 0.0703 + 0.0176 + 0.00195$

$\Rightarrow 0.255803$ //

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Wilcoxon Signed Rank Test

$\mu_T = \frac{n(n+1)}{4}$

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$SD = \sqrt{\frac{n(n+1)(2n+1)}{24}}$ $\quad Z = \frac{T - mean}{SD}$

Q1)

Visual (X)	Y	d = X-Y or Y-X	Rank	+	−
20	19	1	2.5	2.5	
17	16	1	2.5	2.5	
14	15	−1	2.5		−2.5
18	16	2	6	6	
15	13	2	6	6	
16	16	0			
19	15	4	9 → comes directly	9	
16	18	−2	6		−6
17	14	3	8 → comes directly	8	
18	17	1	2.5	2.5	
				$\boxed{36.5}$	$\boxed{8.5}$

from these two minimum will be taken as T

here 1 comes 4 times hence

$\frac{1+2+3+4}{4} = \frac{10}{4} = 2.5$

here 2 comes 3 times hence

$\frac{5+6+7}{3} = \frac{18}{3} = 6$

here 3 comes 1 time

$T = 8.5$

$n = 9$

$\mu_9 = \frac{9(9+1)}{4} = \frac{9 \times \cancel{10}^5}{\cancel{4}2} = \frac{45}{2} = 22.5$

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$SD = \sqrt{\frac{9(9+1)(18+1)}{24}} = \sqrt{\frac{90 \times 19}{24}}$

$SD = \sqrt{\frac{1710}{24}} = \sqrt{71.25} \Rightarrow SD = 8.44$

$z = \frac{T - mean}{SD} = \frac{8.5 - 22.5}{8.44} = \frac{-14}{8.44} = -1.658$

Cal < Tab

Ho is acepted

Ex 10.20 Ten workers were given on-the-job training with a view to shorten their assembly time for a certain mechanism. The result of the time (in minutes) and motion studies before and after the training programme are given below:

Worker	1	2	3	4	5	6	7	8	9	10
Before	61	62	55	62	59	74	62	57	64	62
After	59	63	52	54	59	70	67	65	59	71

Is there evidence that the training programme has shortened the avg assembly time

Sol

Worker	Before	After	d	Rank	+	−
1	61	59	2	2	2	
2	62	63	−1	1		1
3	55	52	3	3	3	
4	62	54	8	7.5	7.5	
5	59	59	0			
6	74	70	4	4	4	
7	62	67	−5	5.5		5.5
8	57	65	−8	7.5		7.5
9	64	59	5	5.5	5.5	
10	62	71	−9	9		9
					22	23

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$\frac{5+6}{2} = \frac{11}{2} = 5.5$

$\frac{7+8}{2} = \frac{15}{2} = 7.5$

$T = 22$

$\mu_T = \frac{n(n+1)}{4} = \frac{9(\cancel{10})}{\cancel{4}2} = \frac{45}{2} = 22.5$

$SD = 8.44$

$z = \frac{22 - 22.5}{8.44} = \frac{-0.5}{8.44} = -0.059$

Ho is accepted

Q1)

Visual (X)	Y	d = x − y	Rank	−	+
125	118	+7	10		10
132	134	−2	2.5	−2.5	
138	130	+8	12.5		12.5
120	124	−4	6	−6	
125	105	+20	15		15
127	130	−3	4	−4	
136	130	+6	8		8
139	132	+7	10		10
131	123	+8	12.5		12.5
132	128	+4	6		6
135	126	+9	14		14
136	140	−4	6	−6	
128	135	−7	10	−10	
127	126	+1	1		1
130	132	−2	2.5	−2.5	
				31	89

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$\frac{2+3}{2} = \frac{5}{2} = 2.5$

$\frac{5+6+7}{3} = 6$

$T = 39$

$\mu = \frac{15 \times 16}{4} = 60$

$SD = \sqrt{\frac{15 \times 16 \times 31}{24}} = \sqrt{310} = 17.60$

$z = \frac{31-60}{17.60} = \frac{-29}{17.60} = -1.6477$

Ho accepted

f test Or Anova

Source of Variance	Sum of squares	Degree of freedom	Mean of squares	Common value of f
B/w the sample	SSB	$c-1$	$MSB = \frac{SSB}{c-1}$	$f = \frac{MSB}{MSW}$
within the same	SSW	$n-c$	$MSW = \frac{SSW}{n-c}$	

ues) The following table gives the yields on 15 sample fields under three varieties of seed (v13 ABC)

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A	B	C
5	3	10
6	5	13
8	2	7
1	10	13
5	0	17
$\bar{X_1} = 5$	$\bar{X_2} = 4$	$\bar{X_3} = 12$

$\bar{\bar{X}} = \frac{(5+4+12)}{3} = \frac{21}{3} = 7$

$\Rightarrow$ SSB = (Sum of squares b/w samples)

A $(\bar{X_1}-\bar{\bar{X}})^2$	B $(\bar{X_2}-\bar{\bar{X}})^2$	C $(\bar{X_3}-\bar{\bar{X}})^2$
$(5-7)^2 = 4$	$(4-7)^2 = 9$	$(12-7)^2 = 25$
$(5-7)^2 = 4$	$(4-7)^2 = 9$	$(12-7)^2 = 25$
$(5-7)^2 = 4$	$(4-7)^2 = 9$	$(12-7)^2 = 25$
$(5-7)^2 = 4$	$(4-7)^2 = 9$	$(12-7)^2 = 25$
$(5-7)^2 = 4$	$(4-7)^2 = 9$	$(12-7)^2 = 25$
20	45	125

$\Rightarrow SSB = 20 + 45 + 125 \Rightarrow SSB = 190$//

$\Rightarrow$ SSW = sum of squares within sample

A $(X_1-\bar{X_1})^2$	B $(X_2-\bar{X_2})^2$	C $(X_3-\bar{X_3})^2$
$(5-5)^2 = 0$	$(3-4)^2 = 1$	$(10-12)^2 = 4$
$(6-5)^2 = 1$	$(5-4)^2 = 1$	$(13-12)^2 = 1$
$(8-5)^2 = 9$	$(2-4)^2 = 4$	$(7-12)^2 = 25$
$(1-5)^2 = 16$	$(10-4)^2 = 36$	$(13-12)^2 = 1$
$(5-5)^2 = 0$	$(0-4)^2 = 16$	$(17-12)^2 = 25$

⇒ $SSW = 26 + 58 + 56$ ⇒ $SSW = 140$//

Source of Variance	Sum of Squares	Degree of freedom	Mean of Squares	Common value of F
B/w the samples	$SSB = 190$	$c-1 = 3-1 = 2$	$MSB = 190/2$, $MSB = 95$	$F = \frac{MSB}{MSW}$
within the samples	$SSW = 140$	$n-c = 15-3 = 12$	$MSW = 140/12$, $MSW = 11.6$	$F = \frac{95}{11.6} = 8.1$

value at degree of freedom $(2, 12) = 3.89 < 8.1$

H_1 accepted

Ques2)

A	B	C
95	93	100
96	98	103
98	92	97
91	100	103
95	90	107
$\bar{x}_1 = 95$	$\bar{x}_2 = 94.6$	$\bar{x}_3 = 102$

$\bar{\bar{x}} = \frac{95 + 94.6 + 102}{3} = \frac{291.6}{3} = 97.2$

⇒ SSB :-

A $(\bar{x}_1 - \bar{\bar{x}})^2$	B $(\bar{x}_2 - \bar{\bar{x}})^2$	C $(\bar{x}_3 - \bar{\bar{x}})^2$
$(95 - 97.2)^2 = 4.84$	$94.6 - 97.2$ ~~$(93 - 94.6)$~~ $= 6.76$	$(102 - 97.2)^2 = 23.04$
$(95 - 97.2)^2 = 4.84$	$(94.6 - 97.2) = 6.76$	$(102 - 97.2)^2 = 23.04$
$(95 - 97.2)^2 = 4.84$	$(94.6 - 97.2) = 6.76$	$(102 - 97.2)^2 = 23.04$
$(95 - 97.2)^2 = 4.84$	$(94.6 - 97.2) = 6.76$	$(102 - 97.2)^2 = 23.04$
$(95 - 97.2)^2 = 4.84$	$(94.6 - 97.2) = 6.76$	$(102 - 97.2)^2 = 23.04$
24.2	33.8	115.2

⇒ $SSB = 24.2 + 33.8 + 115.2 = 173.2$//

⇒ SSW

A $(x_1 - \bar{x}_1)^2$	B $(x_2 - \bar{x}_2)^2$	C $(x_3 - \bar{x}_3)^2$
$(95 - 95)^2 = 0$	$(93 - 94.6) = 2.56$	$(100 - 102) = 4$
$(96 - 95)^2 = 1$	$(98 - 94.6) = 11.56$	$(103 - 102) = 1$
$(98 - 95)^2 = 9$	$(92 - 94.6) = 6.76$	$(97 - 102) = 25$
$(91 - 95)^2 = 16$	$(100 - 94.6) = 29.16$	$(103 - 102) = 1$
$(95 - 95)^2 = 0$	$(90 - 94.6) = 21.16$	$(107 - 102) = 25$
26	71.64	56

⇒ $SSW = 26 + 71.64 + 56 = 153.64$

Source of Variance	Sum of Squares	Degree of freedom	Mean of Square	Common value of f
B/w the samples	$SSB = 173.2$	$c-1 = 3-1 = 2$	$MSB = 173.2/2 = 86.6$	$f = \frac{MSB}{SSW}$
within the samples	$SSW = 153.64$	$n-c = 15-3 = 12$	$MSW = \frac{153.64}{12} = 12.80$	$= \frac{86.6}{12.80} = 6.76$

Two way Classification

Source of variation	Sum of Square	Degree of freedom	Mean squares	Variance ration
B/W Columns	SSC	C-1	MSC $= SSC/(C-1)$	$*F_1 = \frac{\text{Greater variance}}{\text{Smaller variance}}$
within samples	SSR	r-1	MSR = $SSR/(r-1)$	$**F_2 = \frac{\text{Greater variance}}{\text{Smaller variance}}$
Residual/ Error	SSE	(C-1) (r-1)	MSE = $SSE/(C-1)(r-1)$	
Total	SST	rC-1		

Q)

Part of land	A	B	C	Sum
	5	3	10	18
	6	5	13	24
	8	2	7	17
	1	10	13	24
	5	0	17	22
	25	20	60	

$$SSC = \frac{(\text{Sum of sq. of total of each column})}{\text{No. of items in each column}} - \text{Corelation factor}$$

$$SSC = \frac{(25)^2 + (20)^2 + (60)^2}{5} - \text{Corelation factor}$$

$$CF = \frac{T^2}{N} = \frac{(25+20+60)^2}{15} = \frac{(105)^2}{15}$$

$CF = 735$

$SSC = 925 - 735$

$\boxed{SSC = 190}$

$$SSR = \frac{\text{Sum of square of total of each row}}{\text{No. of item in each row}} - \text{Corelation factor (CF)}$$

$$SSR = \left[\frac{(18)^2}{3} + \frac{(24)^2}{3} + \frac{(17)^2}{3} + \frac{(24)^2}{3} + \frac{(22)^2}{3}\right] - 735$$

$SSR = 749.66 - 735.66$

$\boxed{SSR = 14.66}$

$$SST = \text{Sum of square of all the observation} - \text{correlation factor}$$

$SST = 5^2 + 6^2 + 8^2 + 1^2 + 5^2 + 3^2 + 5^2 + 2^2 + 10^2 +$
$10^2 + 13^2 + 7^2 + 13^2 + 17^2 - 735$
$= 1065 - 735$

$\boxed{SST = 330}$

$SSE = SST - (SSC + SSR)$
$330 - (190 + 14.66)$

$\boxed{SSE = 125.34}$

$\Rightarrow c-1 = 3-1 = 2$
$r-1 = 5-1 = 4$
$(c-1)(r-1) = 4 \times 2$
$rc - 1 = 14$

$\Rightarrow MSC = \dfrac{SSC}{c-1} = \dfrac{\cancel{190}^{95}}{2} = 95$

$\Rightarrow MSR = \dfrac{SSR}{(r-1)} = \dfrac{14.66}{4} = 3.66$

$\Rightarrow MSE = \dfrac{SSE}{(c-1)(r-1)} = \dfrac{125.34}{8} = 15.66$

$\Rightarrow f_1 = \dfrac{\text{Greater Var.}}{\text{Smaller Var}} = \dfrac{95}{15.66} = 6.06$
$(2c, 8r)$

$\Rightarrow f_2 = \dfrac{\text{Greater Var}}{\text{Smaller Var}} = \dfrac{15.66}{3.66} = 4.27$
$(8r, 4c)$